Measuring the Mustard Seed: First Exercise in Modelling the Real World

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Constructing, reading and understanding graphs is an interdisciplinary and important skill in today's world. Though being such an important skill students are not taught explicitly to develop this important skill. Also there has been an urge in the literature for the students to use real world data to make sense of the concepts that they learn. In general students are not provided with opportunities to make the skill explicit, and link it up with their life experiences the concepts and subjects that they learn. We present here a simple task which provides the students a context in which real world data is collected, and used to construct a simple linear mathematical model. This task connects different skills like measurement, graphicacy, mathematical modelling and at the same time also helps a two way transition between the abstract mathematical world and the concrete physical world.

Introduction

Mathematical quantitative models are a way to understand the natural world in terms of mathematical symbols and relations. All the models that we have present us with a way to understand, predict and analyse the natural phenomena around us. *A mathematical model is an abstract, simplified, mathematical construct related to a part of reality and created for a particular purpose* (Bender, 1978). Different authors give different interpretations for mathematical modelling and there is some ambiguity in the definitions (Pinker, 1981), but in essence the above definition captures what we mean by a mathematical model in this study. Modelling thus is the hallmark of science. In a particular model the relation between various parameters can be at times quite demanding and complex. We need to present the students with various opportunities to explore the meaning of models in the world around them. Most of the problems students face in textbooks are aimed at giving practice in mathematical manipulations, rather than at giving physical insights, and little scope is provided to them regarding thinking about mathematization of real-life situations (Kapur, 1992).

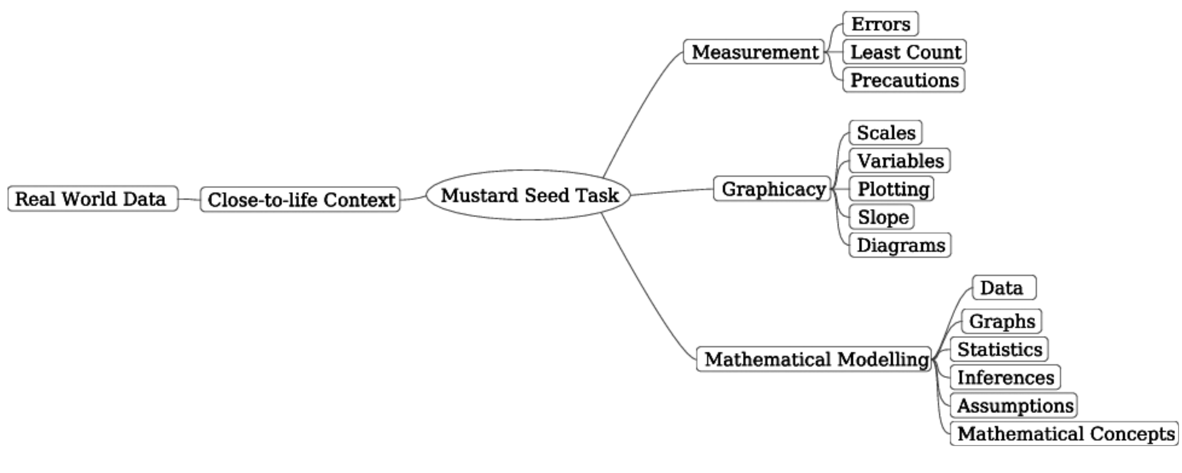

Figure 1. Concepts and related activities for the mustard seed task.

In case of mathematical modelling the students make use of the mathematical concepts that they have learned find patterns and relationships in the data. This requires them to use multiple symbolic systems (tabular, graphical and algebraic in this case) and to understand the relationships between them. Leinhardt et. al suggest that functions and graphs represent one of the earliest points in mathematics in which a student uses one symbolic system to expand and understand the other (Leinhardt, Zaslavsky, & Stein, 1990) This in turn is related to graphicacy, which is defined as the ability to understand and present information in the form of sketches, photographs, diagrams, maps, plans, charts, graphs and other non-textual, two-dimensional formats (Aldrich &

Sheppard, 2000). But if graphs and tables are taught simply as procedural algorithms, then the potential to extract explanations from the patterns and trends observed will be restricted. If children do not understand the purpose of graphs then data handling will not impinge upon the design or the execution of the investigation (Rodrigues, 1994). It has been suggested that school children should be actively involved in collecting "real world" data to construct their own simple graphs. In this way, the application of mathematics to the phenomenon under observation, might enhance students' concept development, particularly operational concepts, correlations, and build and expand the relevant mathematics schemata they need to comprehend the implicit mathematical relationships or theoretical models expressed in graphs (Curcio, 1987). Also, if we take examples from real world to mathematics we can further enhance the development of relevant concepts. This implies encouraging children to design experiments to help them ascertain and determine relationships, not simply teach them the mechanics of graph work (Rodrigues, 1994). Monk suggests that graphing is not a skill which is to be imparted once and for all, but it should be gradually developed across the grades. Graphing must be repeatedly encountered by students as a means of communication and of generating understanding, as the students move across the grades (Monk, 2003). Also in the National Curriculum Framework of 2005 there is a call for emphasis on making connections between mathematics and other subjects. When students learn to draw graphs the functional relationships in science are to be emphasized (NCF, 2005). But in case of Indian textbooks a survey shows a limited use of graphs in the text (Dhakulkar & Nagarjuna, 2011). The graphics in textbooks include pictures, diagrams and graphs, but graphs are rarely used in the textbooks in a way which would provide these relationships.

The problems related to graphicacy are explored in detail by Roth and others in *Critical Graphicacy* (Roth, Pozzer-Ardenghi, & Han, 2005), where the authors address some overarching question about graphicacy: 'What practices are required for reading inscriptions? ' and 'Do textbooks allow students to develop levels of graphicacy required to *critically* read scientific texts? They state "…our aim as critical educators is not just the provision of opportunities for students to become graphically literate; rather, we want students to develop critical graphicacy, that is, we want them to become literate in constructing and deconstructing inscriptions, the deployment of which is always inherently political."

To address the above mentioned issues one of the tasks that we came up with was the measuring of the mustard seed diameter, which forms the basis of this report of an ongoing work. This is part of a larger work on providing students with opportunities to use real world data for constructing, analyzing and developing understanding of the graphs. The task can be seen as a first step towards mathematically modelling more challenging problems from real life situations which have little scope in the standard school texts.

We present here one task for providing the students with opportunities that make this possible. The task is that of measuring the average diameter of the mustard seeds. The objective of the task is to provide students with a context to link up different concepts which span across disciplines and provides the learners with a first hand opportunity to build and test a mathematical model. There is a chance that these concepts would otherwise remain unrelated. The three main concepts as shown in Figure 1 that are linked to the mustard seed task are measurement, graphicacy and mathematical modelling. Each one of these tasks presents its own challenges for the students.

The Indian kitchen is a versatile place where many spices and ingredients mingle to produce a variety of cuisines in different parts of the country. Though each part has a unique style of cooking, there are many things that you will find in each kitchen. One of them is the mustard seed, scientific name *Brassica nigra*. In many of the cuisines the mustard seed is a must, to give a *tadka* or flavour to the food. Thus common availability of the mustard seeds in almost every kitchen provides easy access to the students for doing the measurements. There are two varieties of mustard commonly found in the markets, one variety has seeds almost double size (~2 mm) of the other one (~1 mm).

The size of the seeds is just right enough to make measurements possible with help of a ruler, also they being almost spherical and their easy availability makes them ideal for such experiments. The purpose of this task was to expose the students to to the process of collecting, representing and analysing the data.

The Sample

Most of the students were from the ninth grade, and were selected for a summer camp by the schools themselves, with each school being represented by two students. The medium of instruction at the school for the students was either Marathi or English. Most of the students understood Marathi and Hindi well. Some students had problem in understanding English, so for them the instructions were given in Marathi or Hindi. This was a week long course, in which students were given various activities related to science in general and astronomy in particular. The general aim of the workshop was to introduce students to some aspects of naked eye astronomy and scientific experiments. The workshop included many tasks and tests (pre and post) in which the students participated. One of the tasks that was given to the students was to measure the diameter of the mustard seed. In all there were 4 batches of students, we are presenting the data from batch number one. There were a total of 28 students in this batch.

The Task

All the students in the class were familiar with mustard seeds. The task given to the students was to measure the diameter of the mustard seed. The students were familiar with the properties of a circle (like area and circumference) and a sphere (volume). Two examples which are similar to the mustard seed task involving indirect measurement were discussed in the class. One of the tasks was indirect measurement of width of a thread. This is usually done by winding the thread on a object (for example a pencil) and finding the length for a given number of turns. We would get the average width by dividing this length by the number of turns. The second task that was discussed was to find the thickness of the papers of a given book. This task also involves measuring the length for a given number of papers and then the average thickness is found out by dividing the length by the number of pages. We could use either of these two tasks or the mustard seed task as the approach to modelling.

After these two warm up discussions the students were asked to guess the approximate size of the mustard seeds, for this purpose some mustard seeds were shown to them in the classroom. During the discussions that ensued the students came up with answers like few centimetres to few millimetres.

In the next part of the discussion the students were asked to come up with ideas for measuring the diameter of the seeds. The idea of using a ruler for measuring the diameter was not disclosed to them. Some of the students came up with some ingenious methods of measuring the diameter. One of the students suggested that a thread should be wound on the seed, and then the length of the thread can be measured easily with help of a ruler. Another student suggested an even more elaborate method: we can find out volume of displacement of water due to a seed and then from the volume of the water we can find out the volume of the mustard seed and from the volume we can find out the radius (and hence diameter). The idea that was in general floating around in the discussion was that we have to measure the diameter of only one seed.

To counter this the instructor asked the students to look at the mustard seeds and to tell whether all of them were of exactly same size. To this students responded that they were not of same size. Some of the students responded to this by saying that an average of many values need to be taken. This way the idea of doing multiple measurements and taking averages was brought in.

Then the discussion led to using the task of measuring the diameter of the mustard seed with help of a ruler (1 mm as the least count). In this case the students were told to take measurements of 5, 10, 15, 20, 25 and 30 mustard seeds. The measurement involved aligning the mustard seeds along a scale and measuring the length covered by them. Then the students were to find out the average diameter for each one of the set of seeds. The students were asked to submit a written report on this task. In the report they were told to write the procedure, errors and precautions, assumptions and conclusions for the task. As a final task the students were to also plot a line graph for number of seeds versus the total length that they measured on the ruler. The format of the graphs can have significant impact on how the data is interpreted. Line graphs bring out the trends in the variables under question. If a particular trend is the most important information, then line graphs should be used (Shah, Mayer, & Hegarty, 1999).

Discussions on modelling the data

We tried to build a simple mathematical model from the data that would explain our measurements. Typical of the prevailing descriptions of the modelling process is that given by (Ackoff, 1956), as given in (Pinker, 1981). 1. Formulate the problem; 2. Construct a mathematical model to represent the system under study; 3. Derive a solution from the model; 4. Test the model and the solution derived from it.

The students did the task as a homework exercise and brought in the written reports. As a first step we asked what was the diameter of the mustard seed to each one of them. The answers varied, and it was through some probing on why the answers are varying, sometimes more than double, it emerged that there are in fact two different varieties of mustard seeds. The students had only either of the varieties at their homes, so had the data only for one of them. This provided us with an opportunity to discuss what differences size would mean for the mathematical model that was to be built.

During the discussion in the classroom it was asked to the students whether the values of the length that they have measured for different number of seeds have any mathematical relationship amongst them. First of all it was noted that as the number of seeds increase, the total measured length increases. So it was agreed upon that there is a direct proportion between the length measured for the seeds L and the number of seeds n. So that

$$L \propto n \qquad (1)$$

After the agreement of the two quantities being in direct proportion the discussion was taken further ahead by introducing the constant of proportionality (lets say d) and hence the mathematical relation between the two quantities L, and n can be written as

$$L = d \ ) \ n \qquad (2)$$

At this point the students were reminded of the straight line equation

$$Y = m \ ) \ X \qquad (3)$$

where m is the slope of the line. We then compared the two equations for similar terms. The total length L and the number of seeds n in equation (2) is analogous to the Y and the X values respectively in equation (3). The constant of proportionality d in equation (2) can be seen analogous to the slope m in equation (3). All this leads to the fact that equation (2) is indeed an equation for the straight line.

After showing that the mathematical relationship that is expressed by equation (2) is a straight line, it makes sense as to why we can draw a reasonable straight line passing through all the points. To make the understanding of the slope of the line and its meaning to the mathematical modelling, another exercise was done in the class. In this case the researcher collected the values of length from all the students, and put them on a spreadsheet for the class to see. The values were grouped into two parts, depending on the seeds. Then an average of these collected values was taken.

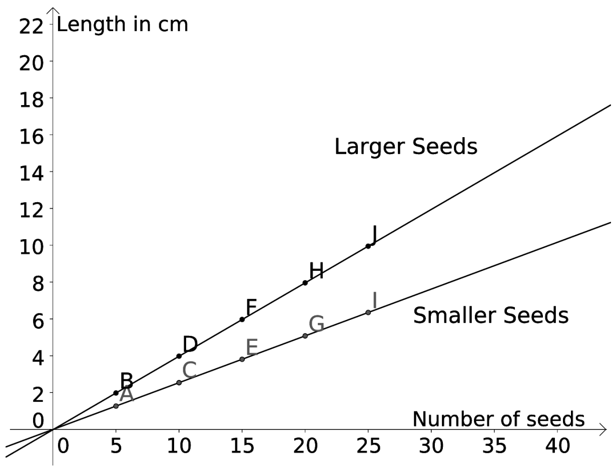

Figure 2: The graphs of lines for two different types of seeds drawn in GeoGebra from the average readings taken from students. The two lines differ in their slopes and this can be linked to the difference in the size of the seeds.

When all the data points for different number of seeds were collected, the researcher plotted these points and fitted a line through the points using GeoGebra [1], see Figure 2. As a result of this exercise, two distinct lines were plotted, corresponding to each type of seed. Discussion followed on what is the physical meaning of the slopes of the line in task at hand. In this case we have a concrete physical observation that the size (diameter) of the two types of seeds is different. On the other hand in the mathematical model, the slope of the lines are different. In this case we can relate the abstract change in the slope of the mathematical form to a concrete observation regarding the size of the mustard seed. This point was discussed at length in the class. Using GeoGebra in the classroom was also helpful to visualise how the lines would have looked like when the slope was different. For example if the seeds had an average diameter of 1 mm or 3 mm, where would the lines be with respect to the lines that were drawn.

The next step in case of mathematical modelling is predicting and testing the model. The questions that can be asked in this regard were can you predict the number of seeds that will form a given length or can you tell the

1 http://geogebra.org

length of given number of seeds. An interesting variation in this would be do this act collaboratively like a quiz. Some of the students may present their data and others are asked to find out about the missing quantities. And when there are discrepancies found in the expected and derived results, the discussions can be fruitful.

Student responses

Out of the 28 students only 2 students did not attempt to draw a graph or write any result. The rest of the students did attempt to draw the graph. In this section we discuss some important trends that we could gather.

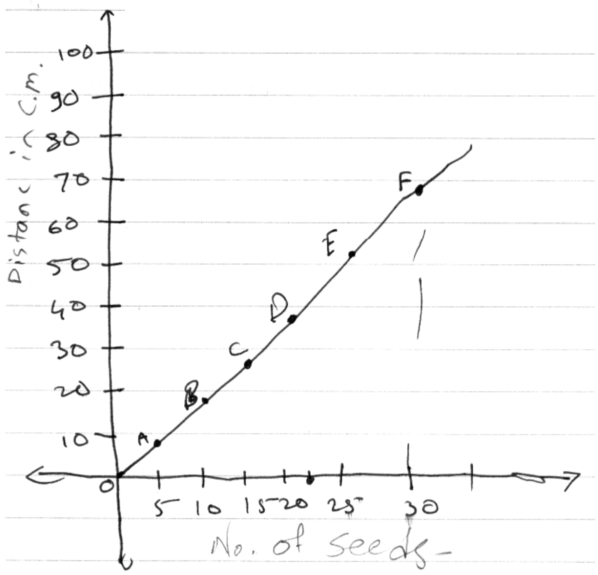

Figure 3. An example of graph from one of the students. Here the Y-axis is labelled as distance in cm, whereas it should be in mm and the points are approximately plotted. The scale on X-axis is not even, and none of the scales are mentioned.

The tables that were used to prepare for the graphs had data in the following format: Number of seeds - Length in mm/cm - Average length for 1 seed. Some of the students used calculators to get accuracy up to 4 decimals, this provided for opportunity to discuss the concept of least count, and number rounding off in the class. Most of the students did the measurements for 5, 10, 15, 20 and 25 seeds, though there were a few students who measured for up to 40 seeds.

Almost all of the students who drew the graph, could plot the data points correctly. Not all of them drew lines through the points that they had plotted. Some of the students drew the graph on the response sheet and not on the graph paper, example Figure 3. Only one student drew both bar graph as well as line graph. Most of the students choose scale of 5 seeds, per unit for X-axis and the scale for the Y-axis was seen to be variable. Many students choose the same scale as the actual readings, with 1 mm on the graph being equal to 1 mm of the actual measurements, example Figure 4. The students were not given specific instructions in choosing the scales, but they were asked to write the scales in the graphs that they drew. One of the students plotted the values of average diameter that was obtained from the measurements against the number of seeds.

During the classroom discussions the possible errors that would occur and ways to eliminate them were discussed. The students were to write about them in the reports. The most common reported error was of the alignment of the seeds with the measuring ruler. This we think was due to the fact that this error was readily perceptible by the students, and they could experience it while performing the task. One of the students actually glued the seeds on the paper to overcome this error! A few students drew diagram of the placement of the seeds with the ruler.

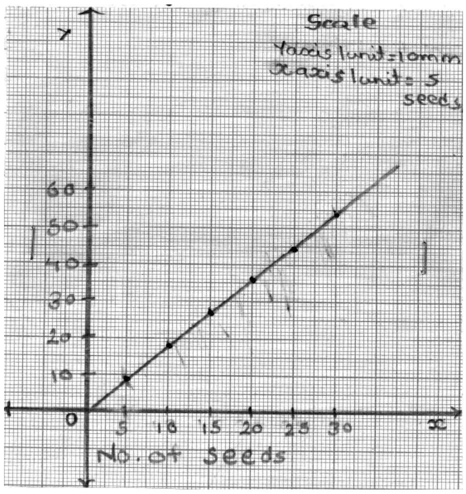

Figure 4. An example of graph from one of the students. Here the Y-axis has scale 1mm = 1mm and X-axis has scale of 5 seeds = 1 cm.

In a mathematical model, it is important to know the assumptions that we have for that model. It has been suggested that for students who are not accustomed to mathematical modelling, the teacher's main role is to clarify assumptions, and to inform students of the importance of assumptions (Seino, 2005). In our case the discussions in the class called for different assumptions that we had while making the measurements and while applying the mathematical model to the data at hand. The students in their responses listed out the assumptions, and sometimes mixed the assumptions with the precautions that they took while performing the task.

Some of the most commonly stated assumptions were [a] the seeds are spherical in shape, [b] the seeds are all of same size [c] they were aligned properly (in a line) while measurement, and [d] the number of seeds was counted properly. Though the two later ones can be said to be more of precautions than assumptions.

Only 8 students in their reports clearly wrote the final result. Many of the students, did only tabular calculations and did not proceed further. In conclusions of the report only three students wrote about the modelling part, and derived the linear equation for mathematical model. This was before the discussions in the class regarding the mathematical model.

Further work

The task of finding the diameter of the mustard can be seen as first step towards modelling of the real world based on data collected by the students themselves. This task though might appear as simple provides rich opportunities for linking modelling, measurement and graphicacy.

Though the task and the model were simple, not all the students could come close to the expected result, shows that the bridging and abstract mathematical knowledge to real world is not trivial. Therefore, it is vital to bring to the classroom such simple tasks, but rich enough to discuss several interrelated concepts in a close-to-life context. Other linear model tasks from real-(world?) such as measuring the thickness of paper, diameter of a thread can be done in continuation to this task. This would emphasise the power of mathematical modelling to the students that using the same general linear model, we can model for systems which are not similar to each other. For example when the points are plotted for all 3 experiments at once, we can understand the associated lengths (diameter/thickness) in terms of the slopes of the lines. Such examples will illustrate the enormous potential and universality of mathematical models to describe the real world. A few such experiments can act as a spring board to scientific modelling, and would help find the links between the models and the real world.

This is an ongoing work and the last part of the study, namely exploring the predictions and testing the model could not be completed in the small time frame that we had for the workshop. For testing the retention of the modelling task and that of predicting and testing the model we plan to conduct further tests and interviews with some of the selected students from the sample.

Acknowledgements

We acknowledge the inspiring Arvind Paranjpye and the interactions with him about the "mustard seed puzzle" that he poses to students. We also appreciate Chetan Thakur and Sujay Mate, summer interns at IUCAA, for their diligent help in data collection. The study would not be possible without the facilities provided by IUCAA.

References

Ackoff, R. L. (1956). The development of operations research as a science. *Operations Research*, *4*(3), 265–295.

Aldrich, F. K., & Sheppard, L. (2000). 'Graphicacy': The fourth 'r'? *Primary Science Review*, *64*, 8–11.

Bender, E. A. (1978). *An introduction to mathematical modeling*. New York: Wiley-Interscience.

Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal of Research in Mathematics Education*, *18*(5), 382–393.

Dhakulkar, A., & Nagarjuna, G. (2011). An analysis of graphs in school textbooks. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of episteme 4 - international conference to review research on science, technology and mathematics education.* India: Macmillan.

Kapur, J. N. (1992). *Insight into mathematical modelling*. New Delhi: Mathematical Sciences Trust Society.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, *60*(1), 1–64.

Monk, S. (2003). A research companion to principles and standards for school mathematics. In (pp. 250–262). NCTM.

NCF. (2005). *National curriculum framework 2005*. New Delhi: NCERT.

Pinker, A. (1981). The concept "model" and its potential role in mathematics education. *International Journal of Mathematical Education in Science and Technology*, *12*(6), 693–707.

Rodrigues, S. (1994). Data handling in the prelimnary classroom: Childrens perception of use of graphs. *Research in Science Education*, *24*, 280–286.

Roth, WM., Pozzer-Ardenghi, L., & Han, J. Y. (2005). *Critical graphicacy (science and technology education library)* (Vol. 26). Netherlands: Springer.

Seino, T. (2005). Understanding the role of assumptions in mathematical modeling: Analysis of lessons with emphasis on "the awareness of assumptions". In P. Clarkson et al.. (Eds.), *Building connections: Theory, research and practice (Proceedings of the 28th annual conference of the mathematics education research group of australasia)* (Vol. 2, pp. 664–671). Sydney: MERGA.

Shah, P., Mayer, R. E., & Hegarty, M. (1999). Graphs as aids to knowledge construction: Signaling techniques for guiding the process of graph comprehension. *Journal of Educational Psychology*, *91*(4), 690–702.